RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022

FIRST YEAR [BATCH 2021-23]

Mathematics

Date : 16/03/2022 **Abstract Algebra 1**

Time : 11.00 am – 1.00 pm **Paper** : MTM CC1 Full Marks:50

Answer all the questions, maximum one can score 50.
(All the symbols have their usual meaning)

1. Let G be a group and H and K be subgroups of G. Show that $(H \cap K)x = Hx \cap Kx$ for all $x \in G$. [3]

2. Let H and K be normal subgroups of a group G. If $H \cap K = \{e\}$, prove that $hk = kh$ for all $h \in H$ and $k \in K$. [3]

3. Show that A_4 has no subgroup of order 6. [5]

4. Let $G = \{a \in \mathbb{R} | -1 < a < 1\}$. Show that $(G, *) \simeq (\mathbb{R}, +)$, where the binary operation $*$ on G is defined by $a * b = \frac{a+b}{1+ab}$ for all $a, b \in G$. [5]

5. Show that there exists (upto isomorphism) only one group of order 77. [7]

6. Find the class equation for S_5. [7]

7. Prove that $I = \{(5n, m) | n, m \in \mathbb{Z}\}$ is a maximal ideal of $\mathbb{Z} \times \mathbb{Z}$. [3]

8. Show that the polynomial $x^2 + \frac{1}{3}x - \frac{2}{5}$ is irreducible in $\mathbb{Q}[x]$. [3]

9. Show that $2x^4 + 6x^3 - 9x^2 + 15$ is irreducible over $\mathbb{Z}$. [2]

10. Show that $\mathbb{Q}[x, y]$ is a UFD, but not a PID. [6]

11. Show that $\mathbb{Z}[i\sqrt{10}]$ is a FD, but not a UFD. [5]

12. In $\mathbb{Z}[i]$, find $\gcd(3 + 5i, 8 + 13i)$. [5]

13. In $\mathbb{Z}_8[x]$, check whether $[1] + [2]x, [4]x + [1]$ and $[4]x + [3]$ are units or not? [6]

– – – – × – – – –